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## ENERGY OF CONDITIONAL INSTABILITY<sup>1</sup>

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The investigations by Margules and by Shaw provide a theoretical foundation for the calculation of the energy of atmospheric instability associated with a given distribution of temperature and humidity:

By the tephigram of Shaw (1), the energy liberated through the ascent of a small element of air of density  $\rho$  in an unstable layer (density  $\rho'$ ), is evaluated; the energy per unit mass is

$$E = \int g \left( \frac{\rho - \rho'}{\rho} \right) dz, \quad (1)$$

in which  $g$  is the acceleration of gravity and  $z$  the height. This evaluation, however, gives no information as to the source from which the released energy is derived; the distribution of temperature and humidity remains unchanged by the thermodynamic process on which the evaluation is based.

Margules (2) calculates the amount of energy freed when a readjustment takes place in an isolated mass of air that initially is in unstable equilibrium. The air is bounded laterally by rigid, nonconducting walls, and above by a weightless piston; the transition to a stable end state takes place through vertical restratification, whereby the piston is raised. Since no external energy is available all the released energy comes from within the closed system considered. Margules calculated, for both the initial unstable condition and the final stable condition, the amounts of potential, internal, and kinetic energy of which the entire energy content is composed, and thus obtains the source of the liberated energy. His fundamental equation is

$$(P + U) - (P' + U') = K + A,$$

where  $P$ ,  $U$ ,  $K$  respectively denote potential, internal, and kinetic energy, and  $A$  is external work.

The results of Margules have clearly shown that energy of instability determined by temperature distribution is of great importance for energy transformations in the atmosphere. Margules' calculations relate almost exclusively to dry air; the results of an extension to humid air differed only slightly from those for dry air. On the basis of these results of Margules, it has come to be generally thought in meteorology that energy of instability in the atmosphere depends essentially only on the temperature distribution, and not on the water vapor content.

From the tephigram, however, Shaw and Refsdal (3) have been led to a different conclusion. Refsdal has

shown that energy of instability also frequently exists in an unsaturated humid atmosphere, even though the stratification is stable with respect to displacements of dry air. According to Shaw and Refsdal, energy of instability materializes from several temperature stratifications only in the presence of water vapor.

This difference arises from the difference between the thermodynamic processes which form the bases of the calculations by Margules and Refsdal. The writer here presents detailed calculations of energy transformations for processes so selected as to include the cases of both Margules and Refsdal. First, the calculations of Margules are extended to conditionally unstable stratifications, and thus the importance of water vapor for the supply of energy clearly shown. Next, the relation between the calculations by Shaw and Refsdal, of the energy for an isolated ascending element, to the calculation by Margules, of the total supply of energy throughout a large closed system, is shown.

### ENERGY SUPPLY IN CONDITIONAL INSTABILITY

To extend the calculations by Margules to conditionally unstable stratifications, we may use essentially the same procedure: A portion of the atmosphere is isolated on all sides by rigid walls, and above by a weightless piston, and this closed system goes from a labile to a stable stratification without exchange of heat with the environment. The amounts of each of the forms of energy are calculated for the initial and the final states, whence from the energy balance one can then determine the source of the energy released during the transition.

In the following example the temperature and the humidity distributions are so selected that in the absence of condensation no energy can be released. Now, by taking account of condensation, however, it is found that energy is set free; and this condition must therefore be attributed entirely to the influence of the water vapor: The initial state is specified by the following values.

Surface pressure,  $p_0 = 1000$  mb; pressure at the upper boundary,  $p_h = 700$  mb; surface temperature,  $T_0 = 296.72^\circ$ ; virtual temperature at the surface,  $T_{0*} = 300^\circ$ ; humidity, 100 percent throughout. The density distribution is so selected that neutral equilibrium obtains as long as no condensation takes place anywhere in the system; that is, the virtual temperature,  $T_v$ , diminishes with the height,  $z$ , at the dry adiabatic rate. Accordingly,  $T_v = T_{0*} - \gamma z$ , where  $\gamma$  is the dry adiabatic gradient.

<sup>1</sup> Translated from the German by Charles M. Lennahan and Edgar W. Woolard.

When condensation is initiated in this atmosphere by vertical motion, air ascending from below can rise to the upper boundary. The system can therefore undergo a transition to a stable state through vertical motions. The final stable state is one in which any element is in stable equilibrium with respect to an arbitrarily great vertical displacement.

In the present example, the transition to the final stable stratification takes place in the following manner. The lower half exchanges place with the upper half. The upper half sinks bodily; but in the ascending half the sequence of layers is reversed, so that the previously lowest elements, initially under the pressure 1000 mb, come to the upper boundary, at a pressure of 700 mb, the elements initially at pressure 900 mb come to pressure 800 mb, and so on. In the final state a lower unsaturated stratum extends from 1000 mb to 850 mb, and an upper super-saturated one from 850 to 700 mb. See figure 1 (plotted on the adiabatic chart of Stüve).

**Potential energy.**—For the distribution of virtual temperature here assumed, the potential energy  $P$  may be calculated in finite form. For any polytropic atmosphere:

$$P = \int g z dm = \int z dp = \frac{T_{0v}}{\gamma} \int_{p_0}^{p_1} \left[ 1 - (p/p_0)^{\frac{R\gamma}{\theta}} \right] dp \\ = \frac{T_{0v}}{\gamma} (p_0 - p_h) - \frac{T_{0v}}{\gamma \left( \frac{R\gamma}{g} + 1 \right)} \left[ p_0 - p_h (p_h/p_0)^{\frac{R\gamma}{\theta}} \right]$$

where  $R$  is the gas constant, and  $\gamma$  the lapse rate.

After the readjustment the air in the upper half contains condensed water in liquid form, as well as water vapor. The density of this air is equal to the sum of the densities of the air, water vapor, and liquid water,  $\rho = \rho_L + \rho_D + \rho_W$ ; we may introduce a generalized virtual temperature, defined by  $T_w = p/R\rho$ , that is the temperature at which dry air would have the density  $\rho$  of the mixture at the same pressure.

In our example, the distribution of density in the final state is such that  $T_w$  in the *upper half* is almost constant. With sufficient accuracy for our purposes, it may be assumed exactly constant between levels of 25 mb pressure difference, and the potential energy of each stratum calculated as follows:

$$P = \int z dp = \int_{p_1}^{p_2} z_1 dp + \frac{R}{g} \int_{p_2}^{p_1} T \log (p_1/p) dp \\ = z_1 (p_1 - p_2) - \frac{RT}{g} [p_1 - p_2 - p_2 \log (p_1/p_2)].$$

In the *lower half* the virtual temperature, and hence the potential energy, remain *unchanged*. If we now calculate the total potential energy in the initial and the final states, we find it to be *greater* in the final state than in the initial; hence the temperature must be higher in the upper half after the readjustment, thus raising part of the mass to a higher level.

This result reveals a fundamental difference between readjustments in unstable dry and humid closed systems: In dry air, energy of instability can be liberated in a restratification only if by this restratification the potential energy is diminished. In humid air, however, energy of instability can also be freed, as this example shows, even though the potential energy in the end condition is greater than at the beginning. These facts are of great importance in connection with the deepening of cyclones.

**Internal energy.**—The internal energy  $U_1$  of dry air at temperature  $T$  is

$$U_1 = c_v T = c_p T - Jpv.$$

Here  $J$  is the mechanical equivalent of heat; and  $p, v$  are the pressure and volume at the temperature  $T$ . The "internal energy" is that energy which depends only on the temperature of the substance considered; the internal energy  $U_w$  of liquid water is accordingly equal to the heat content,  $i = c_{wp} T$ , diminished by the work of expansion during the change in volume from  $0^\circ \text{C.}$  to the temperature  $T$ :

$$U_w = i - Jp(v_{wt} - v_{w0}).$$

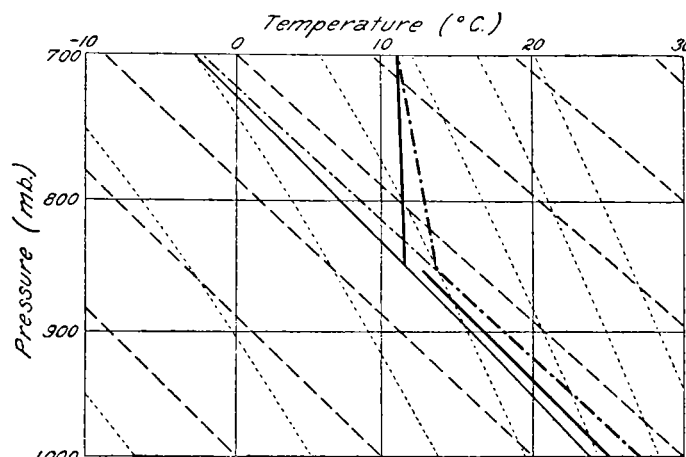


Fig. 1

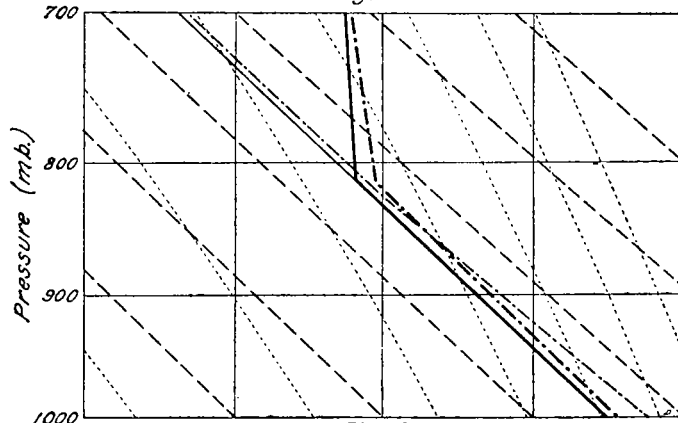


Fig. 2

- Dry adiabats
- ..... Moist adiabats
- Temp. before readjustment
- Temp. after readjustment
- Virtual temp. before adjustment
- Virtual temp. after adjustment

The internal energy of water vapor is equal to the internal energy of the water, plus the internal heat of vaporization, which is the total heat of vaporization  $r$ , diminished by the work of expansion against the partial pressure of the water vapor during the transition from water to vapor:

$$U_d = U_w + r - J e(v_d - v_w), \\ = i - Jp(v_{wt} - v_{w0}) + r - J e(v_d - v_w).$$

However, the quantities  $Jp(v_{wt} - v_{w0})$  and  $Jev_w$  may be neglected, and  $U_d = i + r - Jev_d$  taken as sufficiently accurate.

The above calculated internal energies are for one gram, and must be multiplied by the proportional weights of the

constituents in one gram of moist air, and then integrated over the entire layer, to find the total content of internal energy of the given system.

**External work.**—As the system is bounded laterally by rigid walls, work can be accomplished only at the upper surface when the piston is raised or lowered. The calculation shows that in our example the height of the layer from 1,000 to 700 mb is increased from 2,976.92 m to 3,012.90 m; the work done amounts to  $700 \times 35.98 \text{ m} \times \text{mb}$ , or 251.86 joules.

**Kinetic energy.**—We assume, with Margules, that the system is at rest in the initial state, and that the liberated energy is converted into kinetic energy. We may now calculate the mean wind velocity which the liberated energy can generate in the system.

The results of the energy calculations are shown in table 1. It is apparent that only the water vapor furnishes energy. The complete energy balance is as follows:

Liberated: Internal energy of water vapor.....	Joules 1, 186. 05
Consumed:	
Internal energy of air.....	857. 28
Internal energy of water.....	22. 75
Potential energy.....	17. 70
Work.....	258. 86
	1, 156. 59
Net release.....	29. 46

TABLE I

	Potential energy (joules)	Internal energy (joules)		
		Air	Water-vapor	Water
Initial.....	4, 274. 61	2, 411. 95	7, 624. 59	
Final.....	4, 292. 31	3, 269. 23	6, 438. 54	22. 75
Initial-final.....	-17. 70	-857. 28	+1, 186. 05	-22. 75

Assuming this amount of energy to be uniformly distributed throughout the mass of air, and to appear in the form of kinetic energy, the corresponding mean wind velocity is 13.9 m/sec. (50 km/hr.). The liberated energy comes exclusively from the internal energy of the water vapor.

In dry air the same amount of instability energy would exist if a super-adiabatic gradient of  $1.15^\circ \text{C}/100 \text{ m}$  existed throughout the layer from 1,000 m to 700 m.

#### CALCULATION OF THE ENERGY SUPPLY WITH THE AID OF THE TEPHIGRAM

The usual type of calculation with the tephigram indicates whether energy of instability exists somewhere in the atmosphere and determines the amount which would be freed in the ascent of a small isolated mass of air. We cannot infer from it alone, however, the total energy that would be liberated in a complete readjustment of the system to stable equilibrium; it is necessary to represent the transition of the system as a result of separate steps, and apply the tephigram to each single step. In this procedure, account must be taken of the following three points, which are usually neglected:

(1) In any vertical readjustment, both ascent and descent must take place. The total mass moved is accordingly always about twice as great as that which ascends.

(2) The descending motions may increase or decrease the amount of liberated energy, according as the temperature gradient is greater than or less than the dry adiabatic.

(3) Every partial restratification alters the lapse-rate and the stability, and the system is brought one step nearer a stable state.

The energy in a closed system will now be calculated by both the tephigram and the method of Margules. The initial state selected is one in which the true temperature changes with the pressure at the dry adiabatic rate, the humidity is 70 percent throughout the system, the surface pressure is 1,000 mb, the pressure at the upper boundary is 700 mb, and the surface temperature is  $298.2^\circ \text{A.}$  or  $25.0^\circ \text{C.}$

The final stable state is established by a complete overturn of the system: The air initially at the pressure 1,000 mb comes under the pressure 700 mb and vice versa; and correspondingly, there is an exchange between the other levels that is symmetric about the mean level. See figure 2.

For the purposes of calculation, the system is divided into twelve layers; and the transition to the final state is effected in six stages:

In the first step, the lowest stratum, from 1,000 to 975 mb, exchanges place with the layer from 725 to 700 mb, while the intermediate layers remain in their initial positions; in the second step, the layers from 975 to 950, and from 750 to 725 mb, exchange places; and so on. The energy released in each step is calculated by two methods.

1. The first method of calculation is the same as the one previously used in this paper. The calculation of the potential and the internal energy is repeated after each partial restratification, and an energy balance set up, from which it may be seen how much energy is liberated by this partial restratification, and where this energy has its source. The results are shown in table 2.

TABLE 2

Stage	Exchange	Energy released (joules)	Energy (joule/gr)	Velocity (m/sec.)
1	1,000.....975.....725.....700.....	11. 12	0. 221	21. 1
2	975.....950.....750.....725.....	6. 84	0. 134	16. 4
3	950.....925.....775.....750.....	3. 89	0. 076	12. 6
4	925.....900.....800.....775.....	1. 88	0. 037	8. 6
5	900.....875.....825.....800.....	1. 00	0. 020	6. 3
6	875.....850.....850.....825.....	0. 02	0. 0004	0. 9

Sum, 24.77; mean, 0.810→12.8 m/sec.

2. In the second calculation the tephigram is used; otherwise, the basis is the same as above.

The energy which by equation (1) will be freed in the ascent of the initially lowest element is evaluated in the usual way; in the present example, it amounts to 0.473 joule per gram. In the same way, for an element which sinks from pressure 700 to pressure 1,000 mb, the energy is 0.072 joule per gram. For the particles initially at pressures 975 and 725 mb, the amounts of energy released by the exchange of place are 0.293 and 0.053 joule per gram. Similar calculations are made at pressure steps of 25 mb.

From the values of the energy per gram, the mean values for the 25 mb strata are formed and combined into mean values for the successive stages; these may be compared with the values obtained from the first calculation above, and will be found to agree closely.

The total liberated energy is given by multiplying the value in joules per gram by the mass of air involved. See table 3.

The differences between the amounts of energy for the individual elements and strata are striking. It is not possible to infer the energy supply of an entire layer from

the calculated energy of instability for a particular element.

The first method of calculation leads to an exact energy balance, and shows the source of the liberated energy. The second method of calculation involves less numerical calculation, and gives separately the energy from the ascending and the descending air.

TABLE 3

Pressure (millibars)		Energy released (joule/gr)	Mean (joule/gr)	Mean energy of layer (joule/gr)	Mean velocity (m/sec)
Initial	Final				
1,000	700	0.473	0.272	0.222	21.1
700	1,000	0.072			
975	725	0.293	0.173	0.134	16.4
725	975	0.053			
950	750	0.156	0.096	0.069	11.8
750	950	0.036			
925	775	0.063	0.042	0.028	7.5
775	925	0.021			
900	800	0.017	0.014	0.009	4.3
800	900	0.010			
875	825	0.0028	0.003	0.0014	1.7
825	875	0.0029			
850	850	0.0	0.000		

#### APPLICABILITY OF THE TWO METHODS OF EVALUATION

We shall now consider the problem of which of the processes in the atmosphere correspond to the different methods of evaluation by the tephigram.

The usual method emphasizes the dynamical processes which involve the ascent of a small isolated mass of air; it yields the energy for this mass, but tells nothing about its source. This procedure is appropriate when the stratification of the system is not changed by the displacement of the element; each mass that ascends must be replaced by a mass with the same temperature and humidity, or else enough heat must be supplied to the system to maintain the temperature distribution unaltered (e. g., by continuous radiation). Conditions are most appropriate for the application of this method of evaluation when the vertical equilibrium in the atmosphere is conditionally unstable, but the temperature gradient less than the dry adiabatic; then the equilibrium is stable with respect to downward motion, and the process is one of strong ascending motions over small areas and slow

downward motions over larger areas (4). The energy released will come largely from the ascending elements.

With gradients greater than the dry adiabatic, account must always be taken, even in dry air, of the energy contributions from descending air (5); these can be taken directly from the tephigram, which, therefore, always gives a good indication of the intensity of convection.

The other method of using the tephigram takes into account the changes in the vertical equilibrium brought about by the displacements of air, and should be used whenever the mass of air which ascends is so large that it spreads out into a layer of appreciable thickness. The vertical temperature distribution is then changed, in the absence of a supply of external energy; and the air which subsequently ascends meets with a different environment from that encountered by the previous ascending elements. This method is of fundamental importance to the quantitative determination of the energy of instability for an entire body of air, a knowledge of the amount of such energy is very desirable, because it is a numerical measure of the importance of the body of air for energy transformations in the atmosphere. When the tephigram is used only to calculate energy of instability for a single isolated ascending element, and this is erroneously considered an index to the total available energy, it is easy to overestimate the latter, because in an unstable equilibrium not all the air may ascend to the upper limit of the unstable region and frequently a part of the released energy is taken up by the downward moving air.

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## SYNOPTIC DETERMINATION AND FORECASTING SIGNIFICANCE OF COLD FRONTS ALOFT<sup>1</sup>

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[Soil Conservation Service, Section of Climatic and Physiographic Research, Washington, D. C., December 1936]

Cold-front and warm-front types of occlusions<sup>3</sup> are common on the synoptic chart, but frequently the latter have been confused with the cold-front types. In synoptic analyses the boundaries of the various air masses have all too frequently been considered only from the viewpoint of surface representation, with the result that upper air cold fronts, although occasionally recognized, have generally been held unique. The concept of an upper cold front is by no means new<sup>4</sup>; and recently Wexler<sup>5</sup>

has presented a detailed analysis of a warm-front type of occlusion.

Cold fronts almost invariably become upper-air fronts as a result of warm-front occlusions, although conceivably they may occasionally be generated by fields of frontogenesis with an influence confined to air masses aloft. The recognition of the warm-front type of occlusion and the upper cold front is of paramount significance to meteorologists. The origin of precipitation that occurs throughout the Great Plains in winter may often be directly attributed to an upper cold-front invasion. A forecast of ceilings, cloud layers, thunderstorms, zones of turbulence and icing conditions based upon the recognition of an upper cold front will have distinctive features of immediate pertinency to aircraft travel.

<sup>1</sup> Presented at the meeting of the American Meteorological Society, Kansas City, Missouri, June 1936.

<sup>2</sup> Formerly meteorologist—American Airlines, Inc., Newark, New Jersey.

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